Technical Report

Hydraulic Power Calculation of Francis and Pelton Turbines

1. Introduction

This technical report presents the hydraulic calculations that support the power generation estimates of the turbines in the system proposed by Neon. The system adopts a hybrid configuration combining Francis and Pelton turbines, operating together to maximize the conversion of the water's potential energy into electrical energy.

The calculations were performed based on principles of fluid mechanics and applied hydraulics, taking into account real technical parameters such as head height, flow rate, turbine efficiency, and performance coefficients recommended by the technical literature.

The purpose of this report is to clearly and accurately demonstrate how the hydraulic power of each turbine was determined, providing a solid technical foundation for feasibility analysis by partner companies, investors, and institutions interested in the development and implementation of this sustainable hydroelectric system.

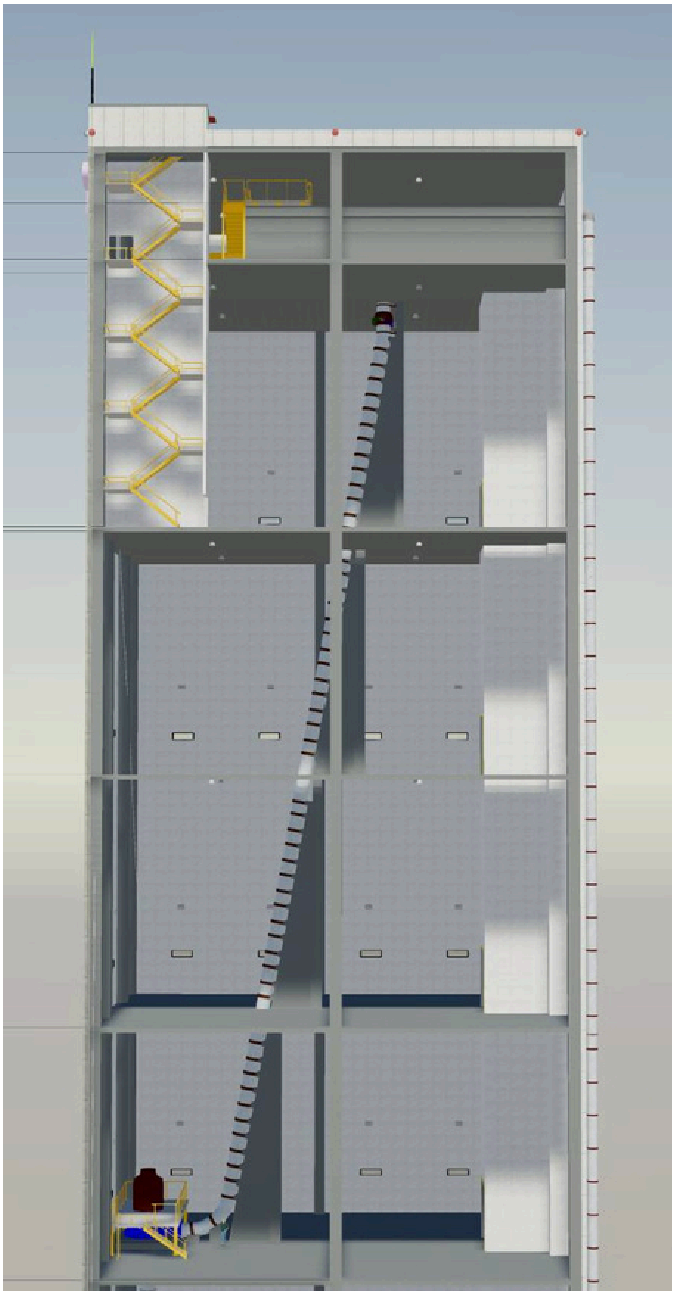
2. System Data

Constants and General Parameters

- Gross head (H): 104.5 m
- Acceleration due to gravity (g): 9.81 m/s²
- Water density (ρ): 1000 kg/m³
- Velocity coefficient (cv): 0.98
- Total system flow rate (Q): 34.84 m³/s
- Pelton turbine efficiency (η_p): 0.90
- Francis turbine efficiency (ηf): 0.80

2.1.Penstock 1

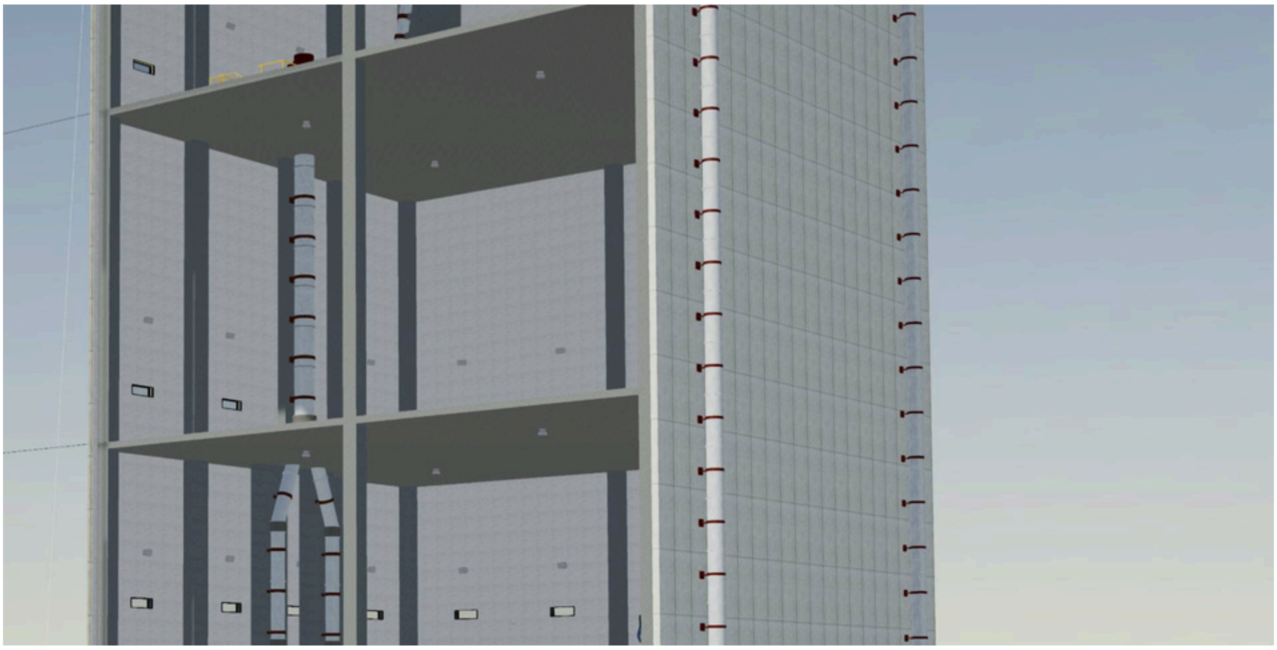
The Penstock 1 is the channel responsible for conducting water from the elevated reservoir to the Francis turbine. It has an internal diameter of 0.90 meters, a radius of 0.45 m, and a cross-sectional area of 0.6362 m².



Internal diameter (D_1): 0.90 m
Radius (r_1): 0.45 m
Cross-sectional area (A_1): 0.6362 m²

2.2. Penstock 2 (Draft Tube – From Francis Turbine to Pelton Nozzle)

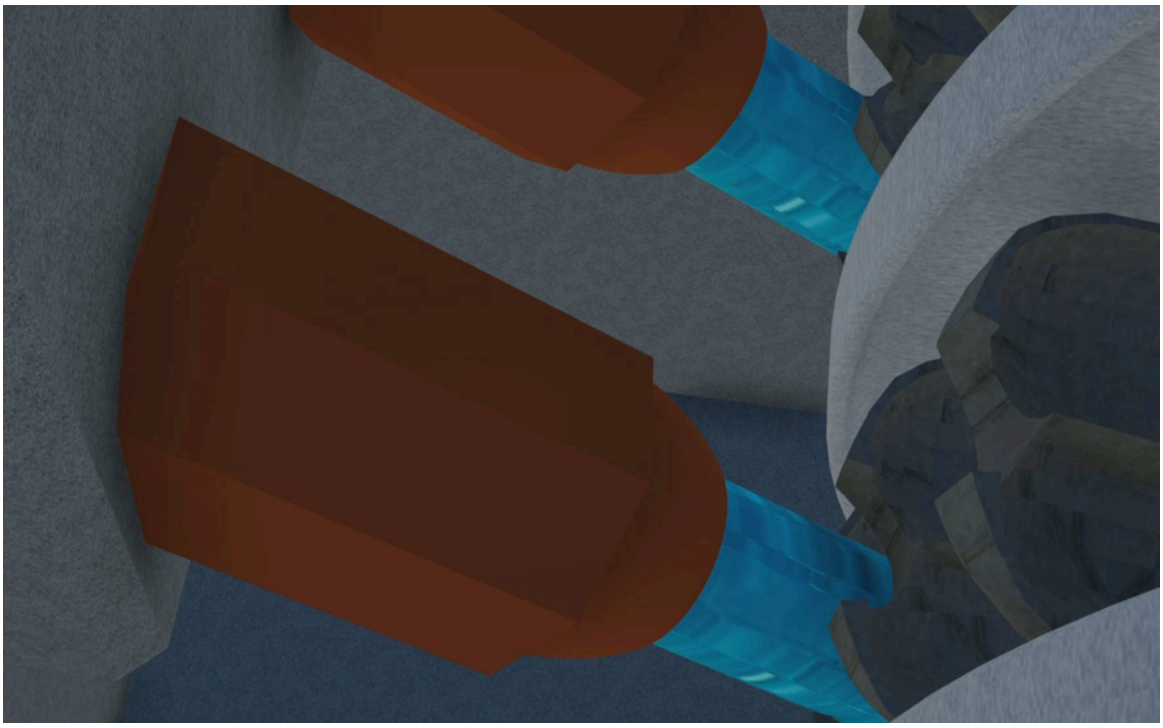
Penstock 2, also known as the draft tube, is the segment that connects the outlet of the Francis turbine to the nozzles that feed the Pelton turbine. It has an internal diameter of 1.00 meter, a radius of 0.5 m, and a cross-sectional area of 0.7854 m².



- Internal diameter (D_2): 1.00 m
- Radius (r_2): 0.50 m
- Cross-sectional area (A_2): 0.7854 m²

2.3. Pelton Turbine Nozzle Dimensions

The nozzles of the Pelton turbine play a key role in converting hydraulic energy into high-intensity kinetic energy. Each nozzle has a diameter of 0.40 meters, resulting in a radius of 0.20 meters and a cross-sectional area of approximately 0.1256 m^2. The system features four symmetrical nozzles, strategically positioned to direct high-velocity water jets with precision onto the Pelton turbine buckets.



- Nozzle diameter: 0.40 m
- Radius (r): 0.20 m
- Cross-sectional area of each jet (A): 0.1256 m^2

3. Water Velocity Calculation in the Penstock

- Head height (H): 104.5 m
- Gravitational acceleration (g): 9.81 m/s²
- Velocity coefficient (cv): 0.98
- V = Water velocity

$$v = cv\sqrt{2 \cdot g \cdot H}$$

$$v = 0,98\sqrt{2 \cdot 9,81 \cdot 104,5}$$

$$v = 0,98\sqrt{2049,09}$$

$$v = \cdot 0,98 \cdot 45,28$$

$$v = 44,37 m/s$$

3.1. Flow Rate Calculation

- Q = Flow rate
- A = Cross-sectional area: 0.7854 m²
- V = Water velocity: 44.37 m/s

$$Q = A \times V$$

$$Q = 0.7854 \times 44.37$$

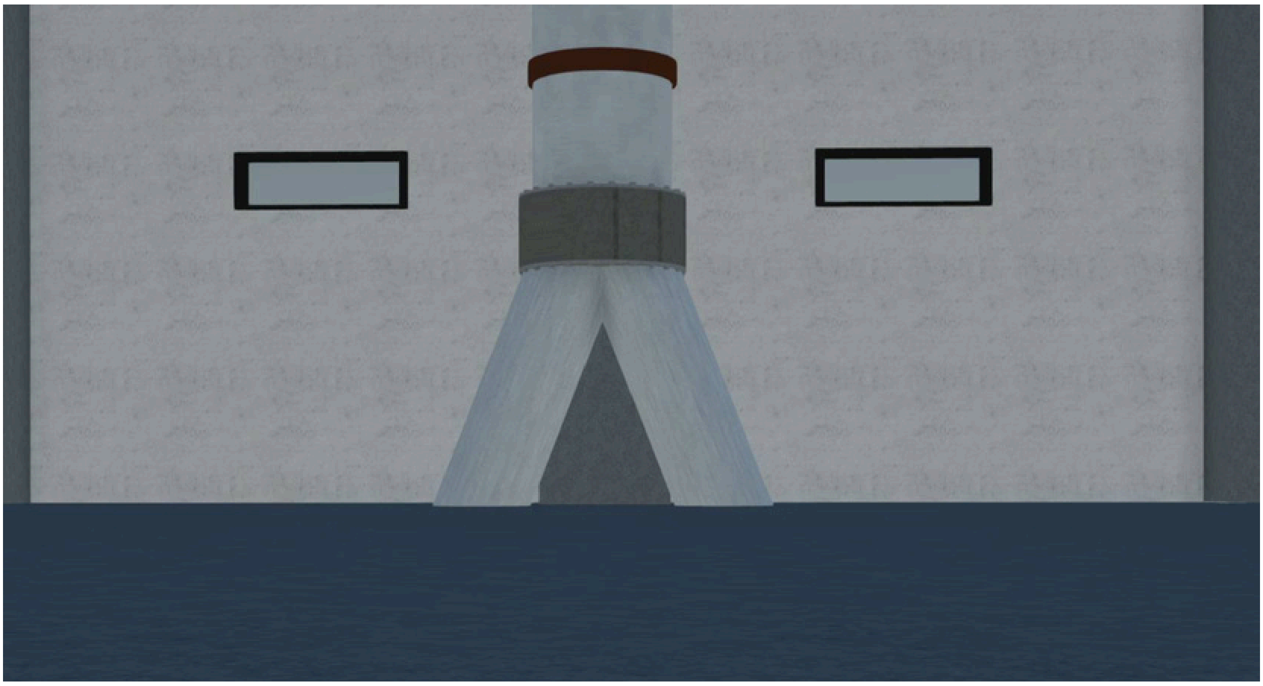
$$Q = 34.84\ m^3/s$$

4. Penstock Division – First Stage

After passing through the Francis turbine, the water flows through a suction pipe known as Penstock 2, which has an initial diameter of 1 meter.

To ensure an efficient and symmetrical flow distribution, this main conduit branches into two identical pipes, constituting the first stage of the flow division process.

- Initial diameter: 1 m
- Initial radius: 0.50 m
- Initial cross-sectional area: A = 0.7854 m^2



4.1. First division: into 2 conduits

$$A1 = \frac{A2}{2} = \frac{0,7854}{2} = 0,3927m^2$$

$$r_1 = \sqrt{\frac{A_1}{\pi}} = \sqrt{\frac{0{,}3927}{\pi}} = \sqrt{0,125} = 0,3535m$$

$$D_1 = 2 \times r_1 = 2 \times 0,3535 = 0,707m$$

$A_1 = 0,3927m^2$

$r_1 = 0,3535m$

$D_1 = 0,707m$

5. Second Stage of Flow Division

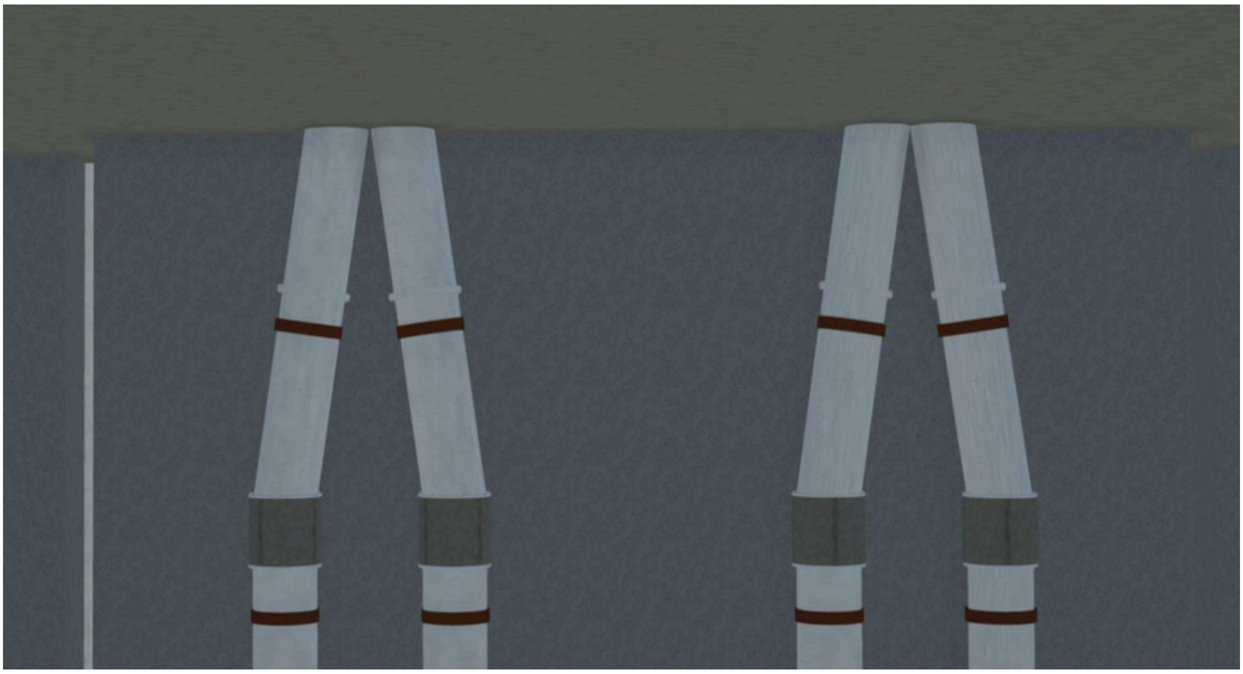
In the second stage of division, each of the two previously split branches divides once more, resulting in a total of four final conduits.

This configuration was adopted to evenly distribute the flow to the four nozzles feeding the Pelton turbine, optimizing energy use and ensuring the hydraulic system's stability.

$D_1 = 0,707m$

$r_1 = 0,3535m$

$A_1 = 0,3927m^2$



5.1.Second Division: into 4 branches

$$A_2 = \frac{A_1}{2} = \frac{0{,}3927}{2} = 0{,}1963 \text{ m}^2$$

$$r_2 = \sqrt{\frac{A_2}{\pi}} = \sqrt{\frac{0{,}1963}{\pi}} = \sqrt{0{,}0625} = 0{,}25m$$

$$D_2 = r_2 \times 2 = 0,25 \times 2 = 0,50m$$

$A_2 = 0,1963m^2$

$r_2 = 0,25m$

$D_2 = 0,50m$

6. Jet Velocity Calculation – Pelton Turbine

After the forced conduit is divided into four symmetrical branches, each branch is directly connected to a nozzle responsible for directing the water jet. These nozzles have a significantly smaller cross-sectional area compared to the penstocks supplying them.

According to the principle of the continuity equation, derived from Bernoulli's theorem, when an incompressible fluid flows from a larger cross-sectional area into a smaller one, its velocity increases proportionally in order to maintain constant flow rate.

This increase in velocity is critical for system efficiency, as it generates four high-pressure water jets that effectively transfer kinetic energy to the Pelton turbine buckets. This direct energy conversion significantly contributes to the high performance of the turbine-generator assembly.

Calculation Parameters:

- Jet radius: r = 0.20 m
- Jet diameter: D = 0.40 m
- Jet cross-sectional area: A = 0.1257 m^2
- Total system flow rate: Q = 34.84 m^3/s
- Number of jets: 4

$$Q_j = \frac{Q_t}{4} = \frac{34,84}{4} = 8,71 m^3/s$$

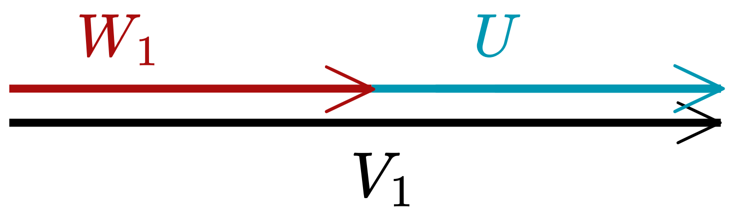
$$Q_j = A_j \times V_j = V_j = \frac{Q_j}{A_{\ j}} = \frac{8,71}{0,1257} = 69,29 m/s$$

$$V_j = 69,29 m/s$$

6.1. Velocity Vector Diagram – Pelton Turbine

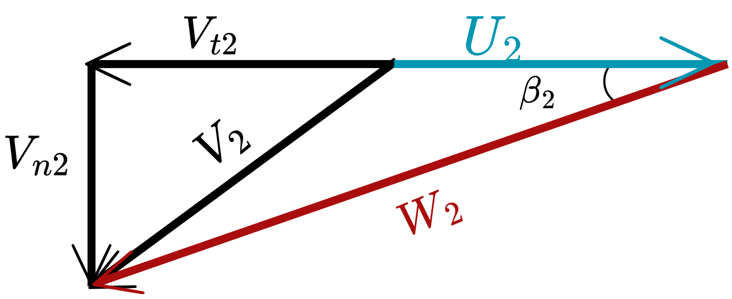
The efficiency of a Pelton turbine reaches its maximum value when the ratio between the tangential velocity of the runner (U) and the absolute velocity of the water jet (V_1) is ideal. In theory, this condition occurs when $\frac{U}{V_1} = \frac{1}{2}$, meaning the tangential velocity is exactly half the jet velocity. This proportion ensures optimal conversion of the jet's kinetic energy into mechanical energy.
In practice, due to mechanical losses and secondary effects such as friction and turbulence, the maximum efficiency occurs at slightly lower ratios, typically between 0.45 and 0.48. In our project, we adopted a value of 0.46.

6.1.1. Velocity Diagram at Inlet



$$U = U_1 = U_2$$
$$U = V_1 \times 0,46$$
$$W_1 = V_1 - U$$

6.1.2. Velocity Diagram at Outlet



$$\beta_2 = 15^0$$
$$V_{t2} = -(W_2 \times Cos\beta_2 - U_2)$$
$$W_2 = W_1 \times K$$
$$0 < K < 1$$

Friction Coefficient: $K = 0,88$

7. Power Calculation – Pelton Turbine

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Fluid Mechanics and Thermodynamics of Turbomachinery

$\rho = 1000kg/m^3$

$Q = 34,84m^3/s$

Absolute Velocity: $V_1 = 69,29m/s$

Tangential Velocity: $U = 31,87m/s$

$U = U_1 = V_1 \times 0,46 = 69,29 \times 0,46 = 31,87m/s$

Relative Velocity at Inlet: $W_1 = 37,42m/m$

$W_1 = V_1 - U = 69,29 - 31,87 = 37,42m/s$

Relative Velocity at Outlet: $W_2 = 32,92m/s$

$W_2 = W_1 \times K = 37,42 \times 0,88 = 32,92m/s$

$Cos\beta_2 = 15^0 = Cos15^0 = 0,965$

Friction Coefficient: $K = 0,88$

Eficiencia global: η=0,90

$$P = \rho \times Q\,(V_1 - U) \times U \times (1 + K \times Cos\beta_2)$$

$$P = 1000 \times 34,84\,(69,29 - 31,87) \times 31,87 \times (1 + 0,88 \times 0,965)$$

$$P = 76.450.761,6W \times \eta$$

$$P = 68.805.685,4W$$

$$P = 68,8MW$$

8. Power Calculation – Pelton Turbine

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$\rho = 1000 kg/m^3$

$Q = 34,84 m^3/s$

Absolute Velocity: $V_1 = 69,29 m/s$

Tangential Velocity: $U = 31,87 m/s$

$U = U_1 = V_1 \times 0,46 = 69,29 \times 0,46 = 31,87 m/s$

Relative Velocity at Inlet: $W_1 = 37,42 m/m$

$W_1 = V_1 - U = 69,29 - 31,87 = 37,42 m/s$

Relative Velocity at Outlet: $W_2 = 32,92 m/s$

$W_2 = W_1 \times K = 37,42 \times 0,88 = 32,92 m/s$

$Cos\beta_2 = 15^0 = Cos 15^0 = 0,965$

Friction Coefficient: $K = 0,88$

Eficiencia global: η=0,90

Tangential Components of the Relative Velocities

$W_{U_1} = W_1 = 37,42 m/s$

$W_{U_2} = W_2 \times Cos\beta_2 = 32,92 \times 0,965 = 31,76 m/s$

$$F = \rho \times Q \times (W_{U_1} - (-W_{U_2}))$$

$$F = 1000 \times 34,84 \times (37,42 + 31,76)$$

$$F = 2.410.231,2 N$$

$$P = F \times U = 2.410.231,2 \times 31,87 = 76.814.068,3 W$$

$$P = 76.814.068,3 \times \eta$$

$$P = 69.132.661,5 W$$

$$P = 69,13 MW$$

9. Power Calculation of the Francis Turbine

The Francis turbine was strategically placed at the intermediate stage of the PMTHPP system to maximize hydraulic performance and ensure stable pressure regulation throughout the cycle.

The first reason for this positioning is the pressure differential that naturally occurs between the turbine's inlet and outlet — a characteristic intrinsic to its design. If not carefully managed, this pressure drop can lead to cavitation and mechanical wear.

The second reason lies in the turbine's function within the pressure recovery process. After transforming gravitational potential energy into mechanical energy, the water flows into a pressurized conduit with a gradually expanding cross-sectional area. This design enables efficient pressure restoration, setting ideal conditions for the next stage of energy conversion.

- Gross Head (H): 80 m
- Flow Rate (Q): 34.84 m^3/s
- Gravity (g): 9.81 m/s^2
- Turbine Efficiency (η): 0.80

$$P = \rho \times Q \times g \times H$$

$$P = 1000 \times 34,84 \times 9,81 \times 80$$

$$P = 27.342.432W$$

$$P = 27.342.432 \times \eta$$

$$P = 21.873.945,6W$$

$$P = 21,87MW$$

10. Power Output of the Francis Turbine in Horsepower

Conversion factor: 1cv=735.5 W

System Parameters:

- Gross Head (H): 80 m
- Flow Rate (Q): 34.84 m^3/s
- Gravitational Acceleration (g): 9.81 m/s^2
- Turbine Efficiency (η): 0.80

$$P = \frac{\rho \cdot Q \cdot H \cdot \eta}{75}$$

$$P = \frac{1000 \cdot 34,84 \cdot 80 \cdot \ 0.8}{75}$$

$$P = 29.730,13 cv$$

$$P = 29.730,13 \times 735,5 = 21.866.513,1 W$$

$$P = 21,87 MW$$

Conclusion – Turbomachinery and Power Generation

This report presented the sizing and performance analysis of the turbines employed in the PMTHPP system, with a focus on hydraulic and energy efficiency within a closed-cycle energy conversion framework.

The Francis turbine was strategically positioned at the intermediate stage of the system to optimize the conversion of gravitational potential energy into mechanical energy while simultaneously increasing the pressure along the descending flow line. This technical choice is justified by both the inherent pressure differential between inlet and outlet—characteristic of Francis turbines—and its role in hydraulic stabilization at the system's midpoint.

The Pelton turbine serves as the system's main power-generating unit, installed horizontally at the base of the hydroelectric structure. Its shaft is symmetrically coupled to two generators—left and right—ensuring optimal dynamic balance and load distribution.

All calculations related to flow rate, velocity, hydraulic power, and efficiency were conducted based on real operating parameters, in accordance with the principles of fluid mechanics and applied thermodynamics. The energy balance confirmed that the energy produced exceeds the energy consumed by the pumps, ensuring the system’s self-sufficiency and providing a surplus for commercial use.

The results obtained validate the technical robustness of the system and confirm its feasibility for application in renewable energy generation projects.