Technical Report

Hydraulic Power Calculation of Francis and Pelton Turbines

1. Introduction

This technical report presents the hydraulic calculations that support the power generation estimates of the turbines in the system proposed by Neon. The system adopts a hybrid configuration combining Francis and Pelton turbines, operating together to maximize the conversion of the water's potential energy into electrical energy.

The calculations were performed based on principles of fluid mechanics and applied hydraulics, taking into account real technical parameters such as head height, flow rate, turbine efficiency, and performance coefficients recommended by the technical literature.

The purpose of this report is to clearly and accurately demonstrate how the hydraulic power of each turbine was determined, providing a solid technical foundation for feasibility analysis by partner companies, investors, and institutions interested in the development and implementation of this sustainable hydroelectric system.

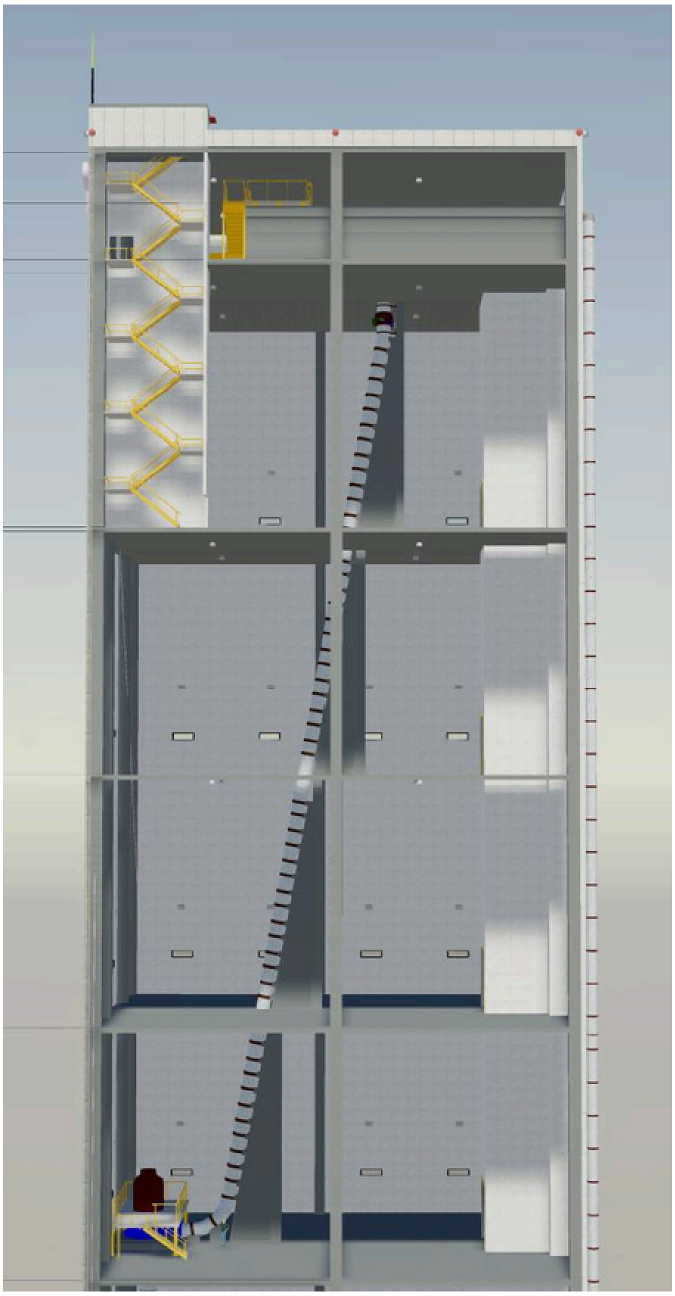
2. System Data

Constants and General Parameters

- Gross head (H): 104.5 m
- Acceleration due to gravity (g): 9.81 m/s^2
- Water density (ρ): 1000 kg/m^3
- Velocity coefficient (cv): 0.98
- Total system flow rate (Q): 34.84 m^3/s

2.1.Penstock 1

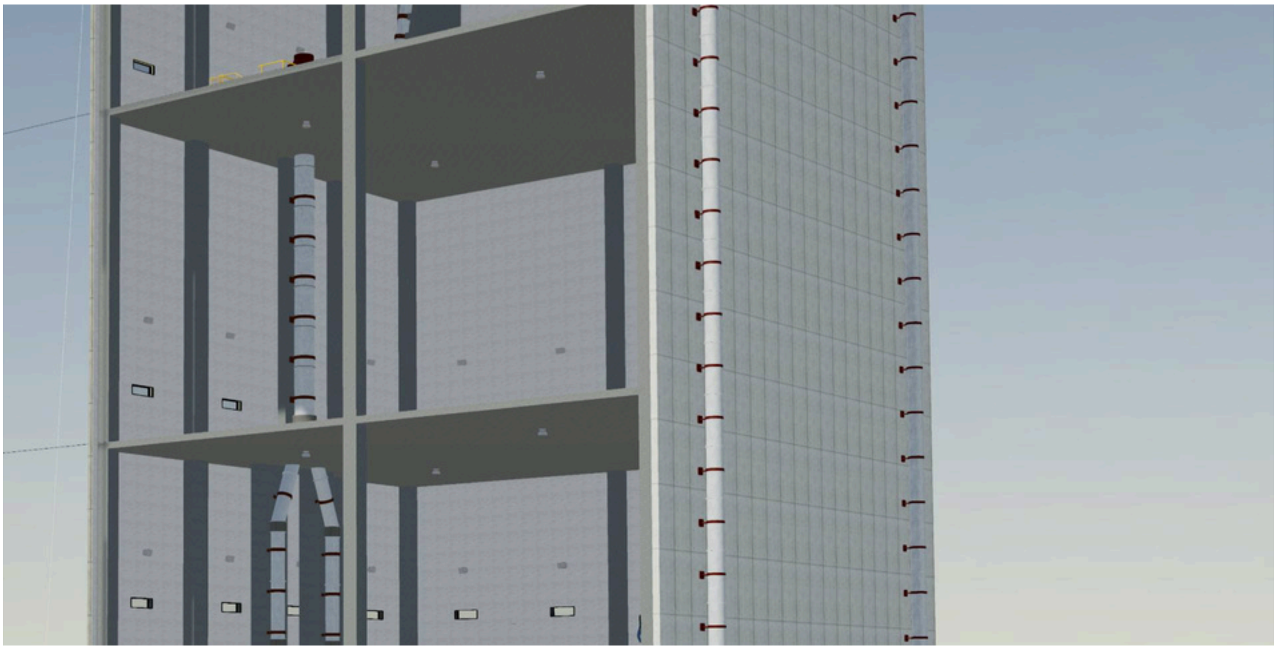
The Penstock 1 is the channel responsible for conducting water from the elevated reservoir to the Francis turbine. It has an internal diameter of2,50 meters, a radius of 1,25m, and a cross-sectional area of 4,909 m^2.



Internal diameter (D_1): 2,50 m
Radius (r_1): 1.25m
Cross-sectional area (A_1): 4,909 m^2

2.2. Penstock 2 (Draft Tube – From Francis Turbine to Pelton Nozzle)

Penstock 2, also known as the draft tube, is the segment that connects the outlet of the Francis turbine to the nozzles that feed the Pelton turbine. It has an internal diameter of 2,70m, a radius of 1,35m, and a cross-sectional area of 5,726 m².



- Internal diameter (D_2): 2,70 m
- Radius (r_2): 1,35 m
- Cross-sectional area (A_2): 5,726 m^2

3. Francis Turbine

The Francis turbine adopted in the PMTHPP system operates similarly to those installed in conventional hydroelectric power plants. Strategically positioned to fully utilize the available gravitational energy, it receives water directly from the upper reservoir, operating with a gross head of 90 meters and a total flow rate of 34.84 m^3/s.

Known for its versatility and high performance, the Francis turbine is designed to maximize the energy extraction from water. As a mixed-flow turbine, its stable and continuous operation contributes directly to the system's operational reliability.

The penstock, with an internal diameter of 2.5 meters, ensures an average flow velocity between 6 and 8 m/s—an ideal range to minimize friction losses, prevent erosion on the pipe walls, and avoid cavitation in the turbine.

After converting potential energy into rotational mechanical energy, the water is directed through a draft tube with an internal diameter of 2.7 meters. This setup allows for partial pressure recovery, reduces the exit velocity, and maintains the system's efficiency at high levels.

In addition to its primary function of power generation, the Francis turbine also serves as a hydraulic transition point within the PMTHPP system. By stabilizing the flow before it reaches the Pelton turbine, it ensures that the downstream flow arrives with optimized hydraulic characteristics, enhancing the performance of the next energy conversion stage.

This synergy between the Francis and Pelton turbines is one of the technical foundations of the PMTHPP project. The integrated configuration allows the system's total energy to be extracted in two consecutive stages, increasing the overall efficiency of the closed-loop cycle and reinforcing the technical and energetic viability of the proposal.

3.1. Power Calculation – Francis Turbine

Gross Head (H): 90 m

Flow Rate (Q): 34.84 m³/s

Gravity (g): 9.81 m/s²

$$\eta_{global} = \eta_h \times \eta_m \times \eta_g$$

$$\eta_{global} = 0,90 \times 0,98 \times 0,97 = 0,85$$

$$P = \rho \times Q \times g \times H \times \eta$$

$$P = 1000 \times 34,84 \times 9,81 \times 90 \times 0,85$$

$$P = 26.146.200W$$

$$P = 26.146,2KW$$

3.2.Power Output of the Francis Turbine in Horsepower

Conversion factor: 1cv=735.5 W

$$P = \frac{\rho \cdot Q \cdot H \cdot \eta}{75}$$

$$P = \frac{1000 \times 34,84 \times 90 \times 0,85}{75}$$

$$P = 35.536,8cv$$

$$P = 35.536,8 \times 735,5 = 26.137.316,4W$$

$$P = 26.137,3KW$$

4. Pelton Turbine

The Pelton turbine is the main power-generating unit of the PMTHPP system. It is installed horizontally at the bottom of the structure, operating with four synchronous turbines in series. Water discharged from the Francis turbine flows into the draft tube and is then channeled through a set of four symmetrical nozzles, each with a diameter of 0.50 meters and a cross-sectional area of 0.1963 m^2, responsible for directing high-speed jets precisely onto the Pelton buckets.

Based on the continuity equation, this geometric configuration ensures the natural acceleration of the flow as it moves from a larger to a smaller section. The result is a set of high-velocity jets, generated within the system's physical limits, efficiently converting the fluid’s kinetic energy into mechanical energy.

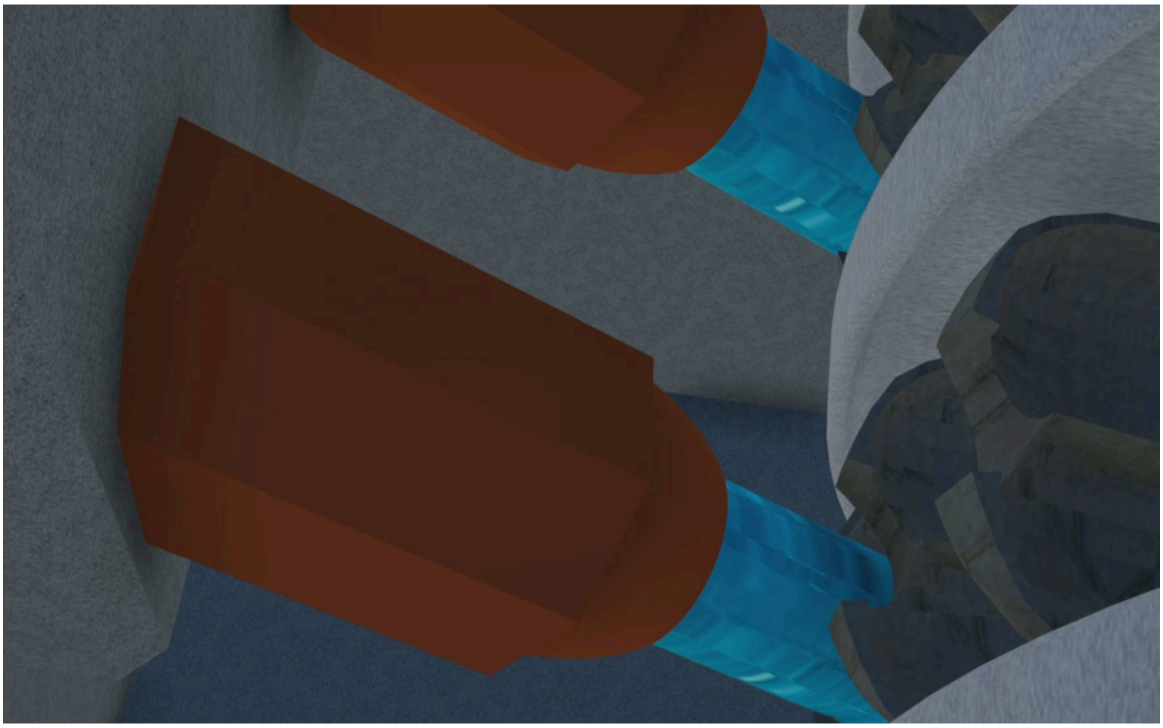
The Pelton turbine operates in a horizontal double-shaft configuration, with two generators symmetrically coupled to the shaft (left and right), allowing for dynamic balance and maximum power output.

Within the closed-cycle structure of the PMTHPP system, the role of the Pelton turbine is to complete the energy conversion initiated by the Francis turbine, ensuring full utilization of the available hydraulic energy.

This final stage consolidates the overall system efficiency, contributing directly to the technical and economic viability of the project.

4.1.Pelton Turbine Nozzle Dimensions

The nozzles of the Pelton turbine play a key role in converting hydraulic energy into high-intensity kinetic energy. Each nozzle has a diameter of 0.50meters, resulting in a radius of 0.25 meters and a cross-sectional area of approximately 0,1963m². The system features four symmetrical nozzles, strategically positioned to direct high-velocity water jets with precision onto the Pelton turbine buckets.



- Nozzle diameter: 0.50 m
- Radius (r): 0.25 m
- Cross-sectional area of each jet (A): 0,1963m²

4.2. Water Velocity Calculation

- Head height (H): 104.5 m
- Gravitational acceleration (g): 9.81 m/s²
- Velocity coefficient (cv): 0.98
- V = Water velocity

$$v = cv\sqrt{2 \cdot g \cdot H}$$

$$v = 0,98\sqrt{2 \cdot 9,81 \cdot 104,5}$$

$$v = 0,98\sqrt{2049,09}$$

$$v = \cdot 0,98 \cdot 45,28$$

$$v = 44,37 m/s$$

4.3. Flow Rate Calculation

- Q = Flow rate
- A = Cross-sectional area: 0.7854 m²
- V = Water velocity: 44.37 m/s

$$Q = A \times V$$

$$Q = 0.7854 \times 44.37$$

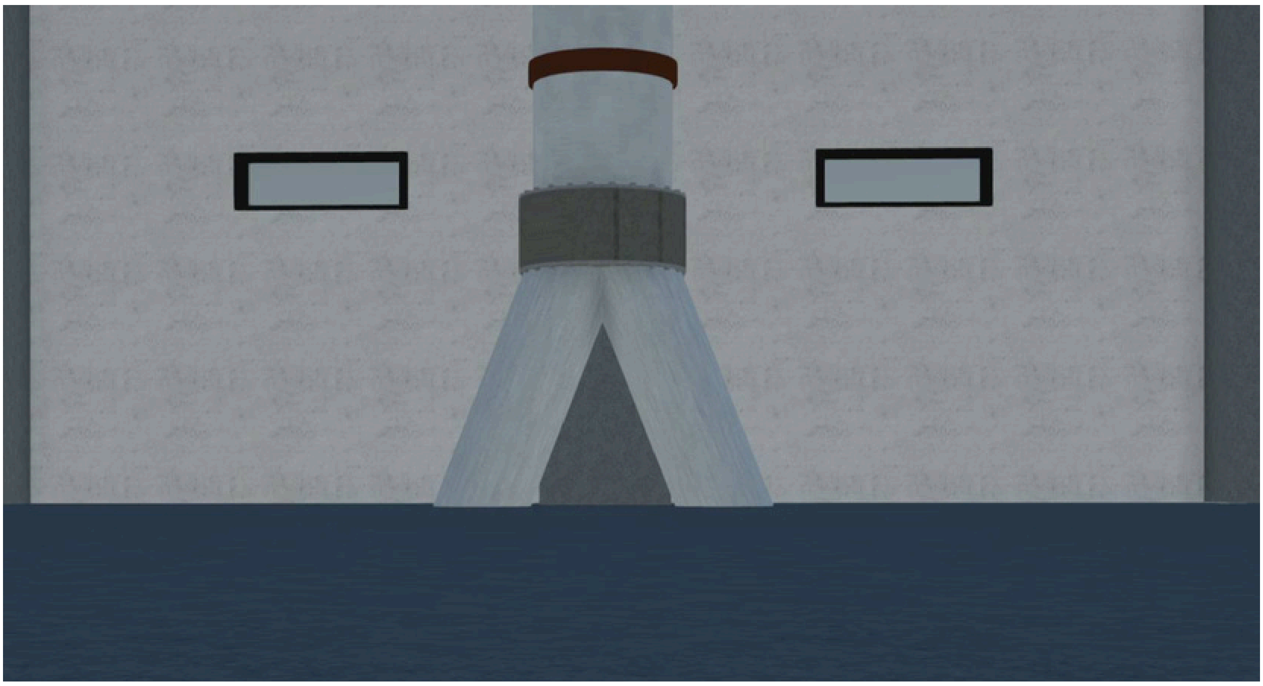
$$Q = 34.84\ m^3/s$$

4.4. Penstock Division – First Stage

After passing through the Francis turbine, the water flows through a suction pipe known as Penstock 2, which has an initial diameter of 2,70 meter.

To ensure an efficient and symmetrical flow distribution, this main conduit branches into two identical pipes, constituting the first stage of the flow division process.

- Initial diameter: 2,70m
- Initial radius: 1,35 m
- Initial cross-sectional area: 5,726m^2



4.5. First division: into 2 conduits

$$A_1 = \frac{A_2}{2} = \frac{5,726}{2} = 2,863m^2$$
$$r_1 = \sqrt{\frac{A_1}{\pi}} = \sqrt{\frac{2,863}{3,14}} = \sqrt{0,91} = 0,954m$$
$$D_1 = 2 \times r_1 = 2 \times 0,954 = 1,90m$$

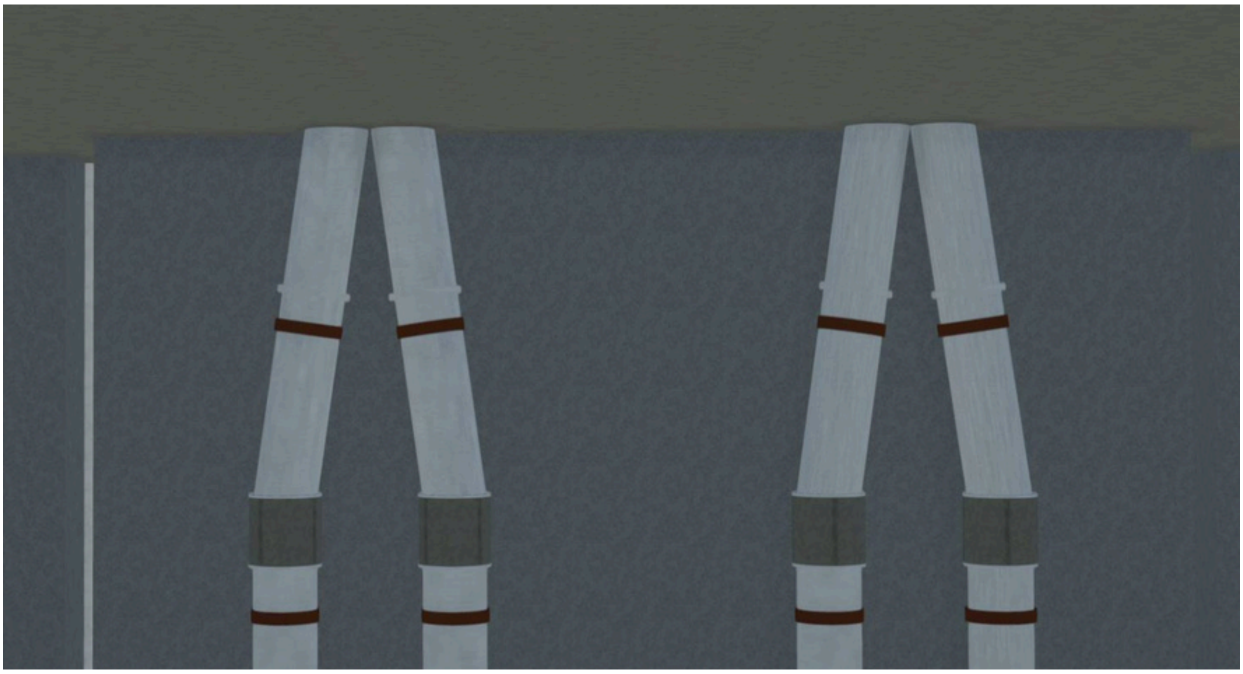
$$A_1 = 2,863m^2$$
$$r_1 = 0,954m$$
$$D_1 = 1,90m$$

4.6. Second Stage of Flow Division

In the second stage of division, each of the two previously split branches divides once more, resulting in a total of four final conduits.

This configuration was adopted to evenly distribute the flow to the four nozzles feeding the Pelton turbine, optimizing energy use and ensuring the hydraulic system's stability.

$$A_1 = 2,863m^2$$
$$r_1 = 0,954m$$
$$D_1 = 1,90m$$



4.7.Second Division: into 4 branches

$$A_2 = \frac{A_1}{2} = \frac{2,863}{2} = 1,431m^2$$

$$r_2 = \sqrt{\frac{A_2}{\pi}} = \sqrt{\frac{1,431}{3,14}} = \sqrt{0,445} = 0,675m$$

$$D_2 = 2 \times r_2 = 2 \times 0,675 = 1,35m$$

$$A_2 = 1,432m^2$$

$$r_2 = 0,675m$$

$$D_2 = 1,35m$$

4.8. Jet Velocity Calculation – Pelton Turbine

After the forced conduit is divided into four symmetrical branches, each branch is directly connected to a nozzle responsible for directing the water jet. These nozzles have a significantly smaller cross-sectional area compared to the penstocks supplying them.

According to the principle of the continuity equation, derived from Bernoulli's theorem, when an incompressible fluid flows from a larger cross-sectional area into a smaller one, its velocity increases proportionally in order to maintain constant flow rate.

This increase in velocity is critical for system efficiency, as it generates four high-pressure water jets that effectively transfer kinetic energy to the Pelton turbine buckets. This direct energy conversion significantly contributes to the high performance of the turbine-generator assembly.

Calculation Parameters:

- Jet radius: r = 0.25 m
- Jet diameter: D = 0.50 m
- Jet cross-sectional area: A = 0,1963 m^2
- Total system flow rate: Q = 34.84 m^3/s
- Number of jets: 4

$$Q_j = \frac{Q_t}{4} = \frac{34,84}{4} = 8,71 m^3/s$$

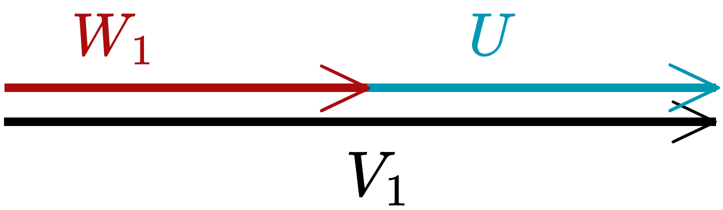
$$V_j = \frac{Q_j}{A_j} = \frac{8,71}{0,1963} = 44,37 m/s$$

$$V_j = 44,37 m/s$$

4.9. Velocity Vector Diagram – Pelton Turbine

The efficiency of a Pelton turbine reaches its maximum value when the ratio between the tangential velocity of the runner (U) and the absolute velocity of the water jet (V_1) is ideal. In theory, this condition occurs when $\frac{U}{V_1} = \frac{1}{2}$, meaning the tangential velocity is exactly half the jet velocity. This proportion ensures optimal conversion of the jet's kinetic energy into mechanical energy.
In practice, due to mechanical losses and secondary effects such as friction and turbulence, the maximum efficiency occurs at slightly lower ratios, typically between 0.45 and 0.48. In our project, we adopted a value of 0.46.

4.10. Velocity Diagram at Inlet

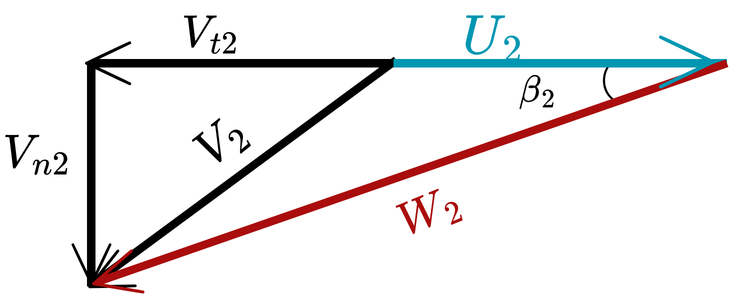


$$U = U_1 = U_2$$

$$U = V_1 \times 0,46$$

$$W_1 = V_1 - U$$

4.11. Velocity Diagram at Outlet



$$\beta_2 = 15^0$$

$$V_{t2} = -(W_2 \times Cos\beta_2 - U_2)$$

$$W_2 = W_1 \times K$$

$$0 < K < 1$$

4.12. Power Calculation – Pelton Turbine

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$\rho = 1000kg/m^3$

$Q = 34,84m^3/s$

Absolute Velocity: $V_1 = 44,37m/s$

$U = U_1 = V_1 \times 0,46 = 44,37 \times 0,46 = 20,41m/s$

Tangential Velocity: $U = 20,41m/s$

$W_1 = V_1 - U = 44,37 - 20,41 = 23,96m/s$

Relative Velocity at Inlet: $W_1 = 23,96m/m$

Relative Velocity at Outlet: $W_2 = W_1 = 23,96m/s$

$Cos\beta_2 = 15^0 = Cos15^0 = 0,965$

$\eta_{global} = \eta_h \times \eta_m \times \eta_g$

$\eta_{global} = 0,93 \times 0,98 \times 0,97 = 0,88$

$P = \rho \times Q\,(V_1 - U) \times U \times (1 + Cos\beta_2)$

$P = 1000 \times 34,84\,(44,37 - 20,41) \times 20,41 \times (1 + 0,965) \times \eta$

$P = 33.464.876W \times 0,88$

$P = 29.449.091W$

$P = 29,449KW$

4.13. Power Calculation Based on Torque

The mechanical power generated at the shaft of the Pelton turbine can be determined by multiplying the transmitted torque by the angular velocity.

4.14. Force Calculation on the Turbine Buckets

$\rho = 1000kg/m^3$

$Q = 34,84m^3/s$

$V_1 = 44,37m/s$

$U = 20,41m/s$

$\beta_2 = 15^0$

$Cos\beta_2 = 0,9659$

$$F = \rho \times Q \times (V_1 - U) \times (1 + Cos\beta_2)$$

$$F = 1000 \times 34,84 \times (44,37 - 20,41) \times (1 + 0,9659)$$

$$F = 1.641.067N$$

4.15. Torque Calculation

$D = 4,6m$

$r = 2,3m$

$$T = F \times r$$

$$T = 1.641.067 \times 2,3$$

$$T = 3.744.454N/m$$

4.16. Mechanical Power Calculation

The useful mechanical power at the turbine shaft output is the direct result of converting hydraulic energy into rotational energy. This power represents the energy available to be transferred to the generator and can be calculated using torque and angular velocity.

$U = 20,41m/s$

$D = 4,6m$

$$U = \frac{\pi \times D \times N}{60}$$

$$N = \frac{U \times 60}{\pi \times D}$$

$$N = \frac{20,41 \times 60}{3,14 \times 4,60}$$

$$N = 84,75RPM$$

$$\omega = \frac{2\pi \times N}{60}$$

$$\omega = \frac{2\pi \times 84,75}{60}$$

$$\omega = 8,87rad/s$$

$$P = T \times \omega$$

$$P = 3.744.454 \times 8,87$$

$$P = 33.213.307W \times 0,88$$

$$P = 29.227,7KW$$

5. Combined Power – Final Result of the Energy Conversion

The PMTHPP system was designed to intelligently and sequentially harness the hydraulic energy available in its closed-loop cycle. The adopted configuration, consisting of two turbines operating in distinct stages, enables a more efficient use of both potential and kinetic energy of the water, thereby maximizing the system's overall efficiency.

The Francis turbine, operating with a 90-meter head and a flow rate of 34.84 m^3/s, generates a net power output of:

$$P_{Francis} = 26,146,2KW$$

The Pelton turbine, installed at the lowest level, contributes with a net power output of:

$$P_{Pelton} = 29.449KW$$

The total net power output generated by the PMTHPP system, already accounting for the actual efficiency of both turbines, is:

$$P_{Total} = 55.595,2KW$$

This value represents the energy effectively available for use, after all mechanical, hydraulic, and thermal losses. The net power generated exceeds the energy consumed by the recirculation pumps, resulting in a measurable energy surplus — essential for the system's self-sufficiency and commercial viability.

Conclusion – Turbomachinery and Power Generation

This report presented the sizing and performance analysis of the turbines employed in the PMTHPP system, with a focus on hydraulic and energy efficiency within a closed-cycle energy conversion framework.

The Francis turbine was strategically positioned at the upper stage of the PMTHPP system to efficiently and safely convert the gravitational potential energy of water into mechanical energy. With a 90-meter head and high flow rate, it operates according to the principles of conventional hydroelectric power plants, ensuring high efficiency.

In addition to generating energy, the Francis turbine stabilizes the flow before it enters the Pelton turbine, ensuring ideal hydraulic conditions for the second stage of the process.

The Pelton turbine serves as the system's main power-generating unit, installed horizontally at the base of the hydroelectric structure. Its shaft is symmetrically coupled to two generators—left and right—ensuring optimal dynamic balance and load distribution.

All calculations related to flow rate, velocity, hydraulic power, and efficiency were conducted based on real operating parameters, in accordance with the principles of fluid mechanics and applied thermodynamics. The energy balance confirmed that the energy produced exceeds the energy consumed by the pumps, ensuring the system's self-sufficiency and providing a surplus for commercial use.

The results obtained validate the technical robustness of the system and confirm its feasibility for application in renewable energy generation projects.